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Class: B.A. Economics (Sem-2nd)
Paper: MJC-2
Topic: Rules of Differentiation
Date: 09-02-2024

The **differentiation rules** help us to evaluate the derivatives of some particular functions, instead of using the general method of differentiation. The process of differentiation or obtaining the derivative of a function has the significant property of linearity. This property makes the derivative more natural for functions constructed from the primary elementary functions, using the procedures of addition and multiplication by a constant number.

The important rules of differentiation are:

- Power Rule
- Sum and Difference Rule
- Product Rule
- Quotient Rule
- Chain Rule

Power Rule of Differentiation

This is one of the most common rules of derivatives. If x is a variable and is raised to a power n, then the derivative of x raised to the power is represented by:

$d/dx(x^n) = nx^{n-1}$

Example: Find the derivative of x^5

Solution: As per the power rule, we know;

$d/dx(x^n) = nx^{n-1}$

Hence, $d/dx(x^5) = 5x^{5-1} = 5x^4$

Sum Rule of Differentiation

If the function is sum or difference of two functions, then the derivative of the functions is the sum or difference of the individual functions, i.e.,

If $f(x)=u(x)\pm v(x)$, then;

$$\boxed{f'(x)=u'(x)\pm v'(x)}$$

Example 1: $f(x) = x + x^3$

Solution: By applying sum rule of derivative here, we have:

$f'(x) = u'(x) + v'(x)$

Now, differentiating the given function, we get;

$f'(x) = d/dx(x + x^3)$

$f'(x) = d/dx(x) + d/dx(x^3)$

$f'(x) = 1 + 3x^2$

Example 2: Find the derivative of the function $f(x) = 6x^2 – 4x$.

Solution:

Given function is: $f(x) = 6x^2 – 4x$

This is of the form $f(x) = u(x) – v(x)$

So by applying the difference rule of derivatives, we get,

$f'(x) = d/dx\ (6x^2) – d/dx(4x)$

$= 6(2x) – 4(1)$

$= 12x – 4$

Therefore, f’(x) = 12x – 4

Product Rule of Differentiation

According to the product rule of derivatives, if the function f(x) is the product of two functions u(x) and v(x), then the derivative of the function is given by:

If f(x) = u(x)×v(x), then:

$$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$$

Example: Find the derivative of $x^2(x+3)$.

Solution: As per the product rule of derivative, we know;

$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$

Here,

$u(x) = x^2$ and $v(x) = x+3$

Therefore, on differentiating the given function, we get;

$f'(x) = d/dx[x^2(x+3)]$

$f'(x) = d/dx(x^2)(x+3)+x^2 d/dx(x+3)$

$f'(x) = 2x(x+3)+x^2(1)$

$f'(x) = 2x^2+6x+x^2$

$f'(x) = 3x^2+6x$

$f'(x) = 3x(x+2)$

Quotient Rule of Differentiation

If f(x) is a function, which is equal to ratio of two functions u(x) and v(x) such that;

f(x) = u(x)/v(x)

Then, as per the quotient rule, the derivative of f(x) is given by;

$$\mathbf{f'(x)} = \frac{\mathbf{u'(x) \times v(x) - u(x) \times v'(x)}}{\mathbf{(v(x))^2}}$$

Example: Differentiate $f(x)=(x+2)^3/\sqrt{x}$

Solution: Given,

$f(x)=(x+2)^3/\sqrt{x}$

$= (x+2)(x^2+4x+4)/\sqrt{x}$

$= [x^3+6x^2+12x+8]/x^{1/2}$

$= x^{-1/2}(x^3+6x^2+12x+8)$

$= x^{5/2}+6x^{3/2}+12x^{1/2}+8x^{-½}$

Now, differentiating the given equation, we get;

$f'(x) = 5/2x^{3/2} + 6(3/2x^{1/2})+12(1/2x^{-1/2})+8(-1/2x^{-3/2})$

$= 5/2x^{3/2} + 9x^{1/2} + 6x^{-½} - 4x^{-3/2}$

Chain Rule of Differentiation

If a function y = f(x) = g(u) and if u = h(x), then the chain rule for differentiation is defined as;

dy/dx = (dy/du) × (du/dx)

This rule is majorly used in the method of substitution where we can perform differentiation of composite functions.

Let's have a look at the examples given below for better understanding of the chain rule differentiation of functions.

Example 1:

Differentiate $f(x) = (x^4 – 1)^{50}$

Solution:

Given,

$f(x) = (x^4 – 1)^{50}$

Let $g(x) = x^4 – 1$ and $n = 50$

$u(t) = t^{50}$

Thus, $t = g(x) = x^4 – 1$

$f(x) = u(g(x))$

According to chain rule,

$df/dx = (du/dt) \times (dt/dx)$

Here,

$du/dt = d/dt\ (t50) = 50t^{49}$

$dt/dx = d/dx\ g(x)$

$= d/dx\ (x^4 – 1)$

$= 4x^3$

Thus, $df/dx = 50t^{49} \times (4x^3)$

$= 50(x^4 – 1)^{49} \times (4x^3)$

$= 200\ x^3(x^4 – 1)^{49}$

Example 2:

Find the derivative of $f(x) = e^{\sin(2x)}$

Solution:

Given,

$f(x) = e^{\sin(2x)}$

Let $t = g(x) = \sin 2x$ and $u(t) = e^t$

According to chain rule,

$df/dx = (du/dt) \times (dt/dx)$

Here,

$du/dt = d/dt\ (e^t) = e^t$

$dt/dx = d/dx\ g(x)$

$= d/dx\ (\sin 2x)$

$= 2 \cos 2x$

Thus, $df/dx = e^t \times 2 \cos 2x$

$= e^{\sin(2x)} \times 2 \cos 2x$

$= 2 \cos(2x)\ e^{\sin(2x)}$